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Constraints on $\tan\beta$ in the MSSM from the upper bound on the mass of the lightest Higgs boson

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Abstract: We investigate the possibilities for constraining $\tan\beta$ within the MSSM by combining the theoretical result for the upper bound on the lightest Higgs-boson mass as a function of $\tan\beta$ with the informations from the direct experimental search for this particle. We discuss the commonly used "benchmark" scenario, in which the parameter values $m_t = 175\,\text{GeV}$ and $M_{\text{SUSY}} = 1\,\text{TeV}$ are chosen, and analyze in detail the effects of varying the other SUSY parameters. We furthermore study the impact of the new diagrammatic two-loop result for m_h, which leads to an increase of the upper bound on m_h by several GeV, on present and future constraints on $\tan\beta$. We suggest a slight generalization of the "benchmark" scenario, such that the scenario contains the maximal possible values for $m_h(\tan\beta)$ within the MSSM for fixed m_t and M_{SUSY}. The implications of allowing values for m_t, M_{SUSY} beyond the "benchmark" scenario are also discussed.

Keywords: Supersymmetric Standard Model, Higgs Physics, LEP HERA and SLC Physics.

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Contents

1. Theoretical basis

Within the MSSM the masses of the $\mathcal{CP}$-even neutral Higgs bosons are calculable in terms of the other MSSM parameters. The mass of the lightest Higgs boson, m_h, has been of particular interest: one-loop calculations [1, 2] have been supplemented in the last years with the leading two-loop corrections, performed in the renormalization group (RG) approach [3]–[6], in the effective potential approach [7, 8] and most recently in the Feynman-diagrammatic (FD) approach [9, 10]. These calculations predict an upper bound on m_h of about $m_h \lesssim 135\,\mathrm{GeV}$.

For the numerical evaluations in this paper we made use of the Fortran code subhpole, corresponding to the RG calculation [5], and of the program FeynHiggs [11], corresponding to the recent result of our FD calculation.

In order to fix our notations, we list the conventions for the input from the scalar top sector of the MSSM: the mass matrix in the basis of the current eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ is given by

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + \cos 2\beta\,(\frac{1}{2} - \frac{2}{3}s_W^2)M_Z^2 & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3}\cos 2\beta\, s_W^2 M_Z^2 \end{pmatrix}, \tag{1.1}$$

where

$$m_t X_t = m_t (A_t - \mu \cot\beta\,)\,. \tag{1.2}$$

For the numerical evaluation, a common choice is

$$M_{\tilde{t}_L} = M_{\tilde{t}_R} =: M_{\mathrm{SUSY}}\,; \tag{1.3}$$

this has been shown to yield upper values for m_h which comprise also the case where $M_{\tilde{t}_L} \neq M_{\tilde{t}_R}$, when M_{SUSY} is identified with the heavier one [10]. We furthermore use the short-hand notation

$$M_S^2 := M_{\mathrm{SUSY}}^2 + m_t^2\,. \tag{1.4}$$

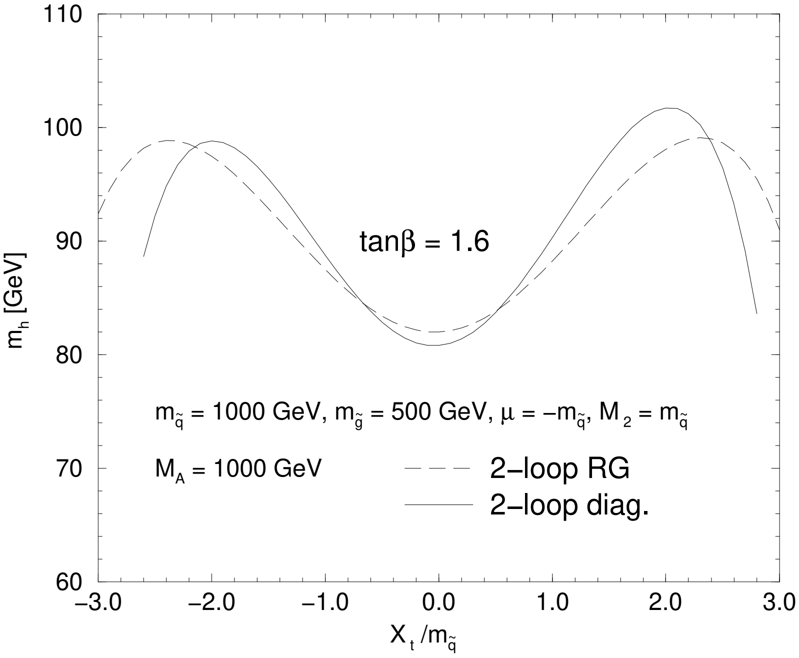

Figure 1: m_h is shown as a function of $X_t/m_{\tilde{q}}$ for $\tan\beta = 1.6$ evaluated in the Feynman-diagrammatic (program FeynHiggs) and in the renormalization group (program subhpole) approach, where $m_{\tilde{q}} \equiv M_{\rm SUSY}$. The maximal value of m_h is obtained for $X_t \approx 2\, m_{\tilde{q}}$ in the FD approach and $X_t \approx 2.4\, m_{\tilde{q}}$ in the RG approach.

While the case $X_t = 0$ is labelled as 'no-mixing', it is customary to to assign 'maximal-mixing' to the value of X_t for which the the mass of the lightest Higgs boson is maximal. As can be seen in figure 1, where m_h is shown as a function of $X_t/M_{\rm SUSY}$ within the FD and the RG approach, the 'maximal-mixing' case corresponds to $X_t \approx 2\, M_{\rm SUSY}$ in the FD approach, while it corresponds to $X_t = \sqrt{6}\, M_{\rm SUSY} \approx 2.4\, M_{\rm SUSY}$ in the RG approach. It should be noted in this context that, due to the different renormalization schemes utilized in the FD and the RG approach, the (scheme-dependent) parameters X_t and $M_{\rm SUSY}$ have a different meaning in the two approaches, which has to be taken into account when comparing the corresponding results. While the resulting shift in $M_{\rm SUSY}$ turns out to be small, sizable differences occur between the numerical values of X_t in the two schemes, see refs. [10, 12].

The main differences between the RG and the FD calculation have been investigated in refs. [12, 13]. They arise at the two-loop level. The dominant two-loop contribution of $\mathcal{O}(\alpha\alpha_s)$ to m_h^2 in the FD approach reads:

$$\begin{aligned}
\Delta m_h^{2,\alpha\alpha_s} &= \Delta m_{h,\log}^{2,\alpha\alpha_s} + \Delta m_{h,{\rm non-log}}^{2,\alpha\alpha_s}\,, \\
\Delta m_{h,\log}^{2,\alpha\alpha_s} &= -\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_t^4\left[3\log^2\left(\frac{\overline{m}_t^2}{M_S^2}\right) + 2\log\left(\frac{\overline{m}_t^2}{M_S^2}\right) - 3\frac{X_t^2}{M_S^2}\log\left(\frac{\overline{m}_t^2}{M_S^2}\right)\right]\,, \\
\Delta m_{h,{\rm non-log}}^{2,\alpha\alpha_s} &= -\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_t^4\left[4 - 6\frac{X_t}{M_S} - 8\frac{X_t^2}{M_S^2} + \frac{17}{12}\frac{X_t^4}{M_S^4}\right]\,; \qquad (1.5)
\end{aligned}$$

therein $\overline{m}_t$ denotes the running top-quark mass

$$\overline{m}_t \equiv \overline{m}_t(m_t) \approx \frac{m_t}{1 + \frac{4}{3\,\pi}\alpha_s(m_t)}\,. \tag{1.6}$$

By transforming the FD result into the $\overline{\mathrm{MS}}$ scheme, it has been shown analytically that the RG and the FD approach agree in the logarithmic terms [12]. The non-logarithmic terms $\Delta m^{2,\alpha\alpha_s}_{h,\mathrm{non-log}}$, however, are genuine two-loop terms, obtained by explicit diagrammatic calculation [12, 13]. In the maximal-mixing scenario, these terms can enhance the lightest Higgs-boson mass by up to 5 GeV (see also the discussion of figure 3 and the corresponding footnote.)

The new two-loop terms obtained within the FD approach lead to a reduction of the theoretical uncertainty of the Higgs-mass prediction due to unknown higher-order corrections (see ref. [12] for a discussion). Another source of theoretical uncertainty is related to the experimental errors of the input parameters, such as m_t. In the case of the SUSY parameters, direct experimental information is lacking completely. For this reason it is convenient to discuss specific scenarios, where certain values of the parameters are assumed.

2. The benchmark scenario

In recent years it has become customary to discuss the restrictions on $\tan\beta$ from the search for the lightest Higgs boson within the so-called "benchmark" scenario, which is specified by the parameter values

$$m_t = 175\,\mathrm{GeV}\,, \qquad M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}\,, \tag{2.1}$$

where M_{SUSY} denotes the common soft SUSY breaking scale for all sfermions (see e.g. refs. [14, 15, 17, 16, 18] for recent analyses within this framework). According to refs. [14, 18, 19, 20], the other SUSY parameters within the benchmark scenario are chosen as

$$\begin{aligned} \mu &= -100\,\mathrm{GeV} \\ M_2 &= 1630\,\mathrm{GeV} \\ M_A &\leq 500\,\mathrm{GeV} \\ A_t &= 0 \quad (\text{"no mixing"}) \\ A_t &= \sqrt{6}\,M_{\mathrm{SUSY}} \quad (\text{"maximal mixing"}), \end{aligned} \tag{2.2}$$

where μ is the Higgs mixing parameter, M_2 denotes the soft SUSY breaking parameter in the gaugino sector, and M_A is the $\mathcal{CP}$-odd Higgs-boson mass. The maximal possible Higgs-boson mass as a function of $\tan\beta$ within this scenario is obtained

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for $A_t = \sqrt{6}\,M_{\rm SUSY}$ and $M_A = 500\,\mathrm{GeV}$. Exclusion limits on $\tan\beta$ within this scenario follow by combining the information from the theoretical upper bound in the $\tan\beta$-m_h plane with the direct search results for the lightest Higgs boson.

The tree-level value for m_h within the MSSM is determined by M_A, $\tan\beta$ and the Z-boson mass M_Z. Beyond the tree-level, the main correction to m_h stems from the t-$\tilde{t}$-sector. Thus, the most important parameters for the corrections to m_h are m_t, $M_{\rm SUSY}$ and X_t.

Since the benchmark scenario relies on specifying the two parameters $m_t = 175\,\mathrm{GeV}$ and $M_{\rm SUSY} = 1\,\mathrm{TeV}$, it is of interest to investigate whether the other inputs in the benchmark scenario are allowed to vary in such a way that the maximal possible value for m_h, once m_t and $M_{\rm SUSY}$ are fixed, is contained in this scenario. This is however not the case:

- Compared to the "benchmark" value of $M_2 = 1630\,\mathrm{GeV}$, the value of m_h is enhanced by about $2.5\,\mathrm{GeV}$ (depending slightly on the value of $\tan\beta$) by choosing a small value for M_2, e.g. $M_2 = 100\,\mathrm{GeV}$ (see ref. [10], where a scan over the MSSM parameter space has been performed showing that the maximal values for m_h are obtained for small values of M_2 and $|\mu|$).

- While in the benchmark scenario only M_A values up to $500\,\mathrm{GeV}$ are considered, higher M_A values lead to an increase of m_h. For $M_A = 1000\,\mathrm{GeV}$, m_h is enhanced by up to $1.5\,\mathrm{GeV}$.

- While within the benchmark scenario "maximal mixing" is defined as

$$A_t = X_t + \mu\cot\beta \; = \sqrt{6}\,M_{\rm SUSY}\,, \tag{2.3}$$

  the maximal Higgs-boson masses are in fact obtained (in the RG approach) for

$$X_t = \sqrt{6}\,M_{\rm SUSY} \quad (\mathrm{RG})\,. \tag{2.4}$$

  This changes m_h by $\mathcal{O}(300\,\mathrm{MeV})$ for $\tan\beta = \mathcal{O}(1.6)$ and $\mu = -100\,\mathrm{GeV}$. As mentioned above, in the FD calculation one has to take

$$X_t = 2\,M_{\rm SUSY} \quad (\mathrm{FD}) \tag{2.5}$$

  for maximal mixing.[1]

[1] As already explained in section 1, the different values for X_t yielding the maximal m_h values in the FD and in the RG approach reflect the fact that this (unobservable) parameter has a different meaning in both approaches due to the different renormalization schemes employed. This has been analyzed in detail in ref. [12]. Thus using different X_t values in the FD and the RG calculation takes this scheme difference into account and individually maximizes the m_h values, see figure 1.

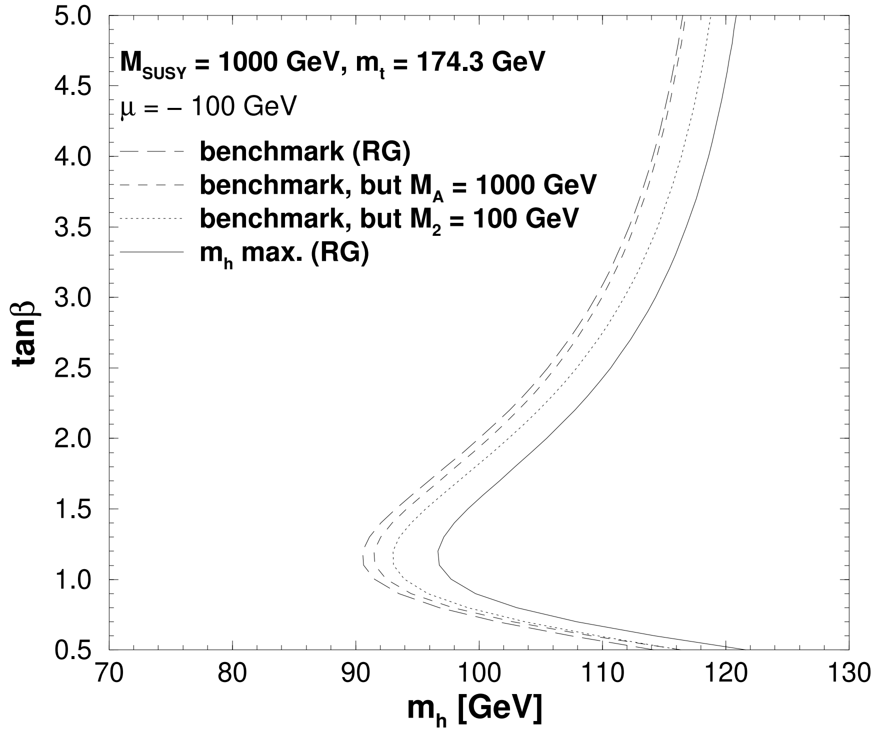

Figure 2: m_h is shown as a function of $\tan\beta$, evaluated in the RG approach. The left (long-dashed) curve displays the benchmark scenario. For the dotted (dashed) curves one deviation from the benchmark scenario, $M_2 = 100\,\mathrm{GeV}$ ($M_A = 1000\,\mathrm{GeV}$), is taken into account. The solid curve displays the maximal possible m_h value for $m_t = 174.3\,\mathrm{GeV}$ and $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$.

- In the benchmark scenario, according to the implementation in the HZHA event generator [19], the running top-quark mass has been defined by including corrections up to $\mathcal{O}(\alpha_s^2)$. Compared to the definition (1.6), which includes only corrections up to $\mathcal{O}(\alpha_s)$, this leads to a reduction of the running top-quark mass by about 2 GeV. From the point of view of a perturbative calculation up to $\mathcal{O}(\alpha\alpha_s)$ it is however not clear whether corrections of $\mathcal{O}(\alpha_s^2)$ in the running top-quark mass, which is inserted into an expression of $\mathcal{O}(\alpha)$, will in fact lead to an improved result. On the contrary, as a matter of consistency of the perturbative evaluation it appears to be even favorable to restrict the running top-quark mass to its $\mathcal{O}(\alpha_s)$ expression (1.6). Adopting this more conservative choice leads to an increase of m_h by up to 1.5 GeV.

All four effects shift the Higgs-boson mass to higher values. For the analyses below we will use the current experimental value for the top-quark mass, $m_t = 174.3\,\mathrm{GeV}$ [22], i.e. we consider the benchmark scenario with $m_t = 174.3\,\mathrm{GeV}$ and $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$. Two of the effects discussed above are displayed in figure 2, where also the maximal values for m_h according to the discussion above ($m_h^{\max}$-scenario: $M_2 = 100\,\mathrm{GeV}$,

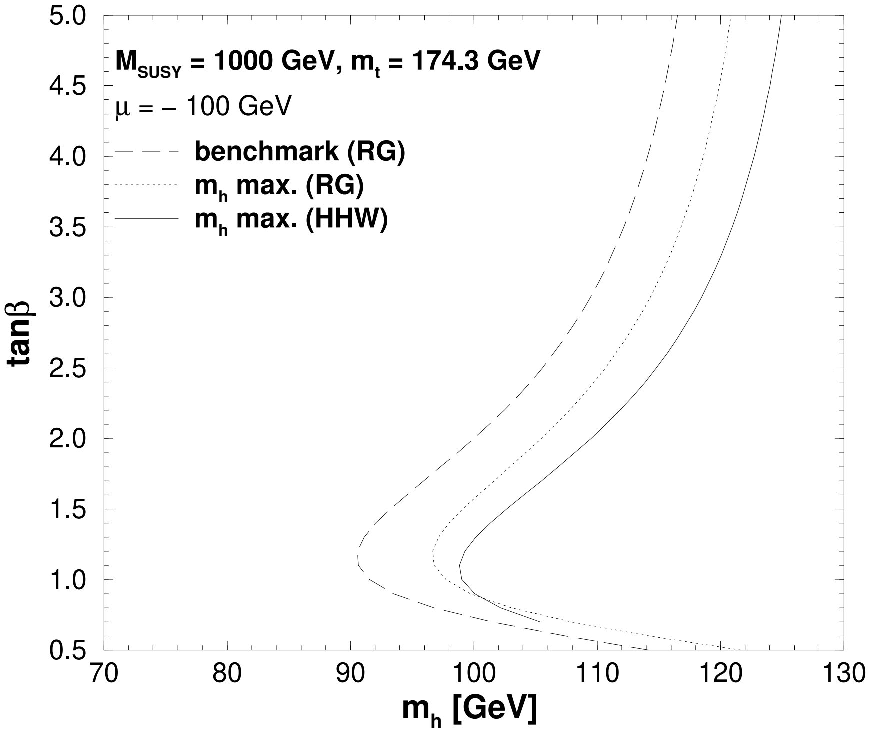

Figure 3: m_h is shown as a function of $\tan\beta$. The dashed curve displays the benchmark scenario. The dotted curve shows the $m_h^{\max}$-RG scenario (program subhpole), while the solid curve represents the $m_h^{\max}$-FD scenario (HHW, program FeynHiggs).

$M_A = 1000\,\text{GeV}$, $X_t = \sqrt{6}\,M_{\text{SUSY}}$ (RG), $X_t = 2\,M_{\text{SUSY}}$ (FD), $\overline{m}_t$ as defined in eq. (1.6)) obtained in the RG approach with $m_t = 174.3\,\text{GeV}$ and $M_{\text{SUSY}} = 1\,\text{TeV}$ are displayed. Comparing the $m_h^{\max}$-scenario with the benchmark scenario, the values for m_h are higher by about $5\,\text{GeV}$.

So far we have only discussed the increase in the maximal value of the Higgs-boson mass which is obtained using the slight generalization of the benchmark scenario discussed above. Now we also take into account the impact of the new FD two-loop result for m_h, which contains previously unknown non-logarithmic two-loop terms. The corresponding result in the $\tan\beta$-m_h plane (program FeynHiggs) is shown in figure 3 in comparison with the benchmark scenario and the $m_h^{\max}$-RG scenario (program subhpole). The maximal value for m_h within the FD result is higher by up to $4\,\text{GeV}$ compared to the $m_h^{\max}$-RG scenario[2] and by up to $9\,\text{GeV}$ compared to the benchmark scenario.

[2] In ref. [12] it has recently been shown that (in the leading m_t^4 corrections to m_h) a large part of the genuine two-loop corrections included in the FD calculation can be absorbed by an appropriate scale choice of the running top-quark mass into an effective one-loop result. Modifying the RG result by using this scale choice for the running top-quark mass would lead to an increase of the RG curve in figure 3 by up to $3\,\text{GeV}$, leaving only a difference of 1–2 GeV between the FD and the RG result.

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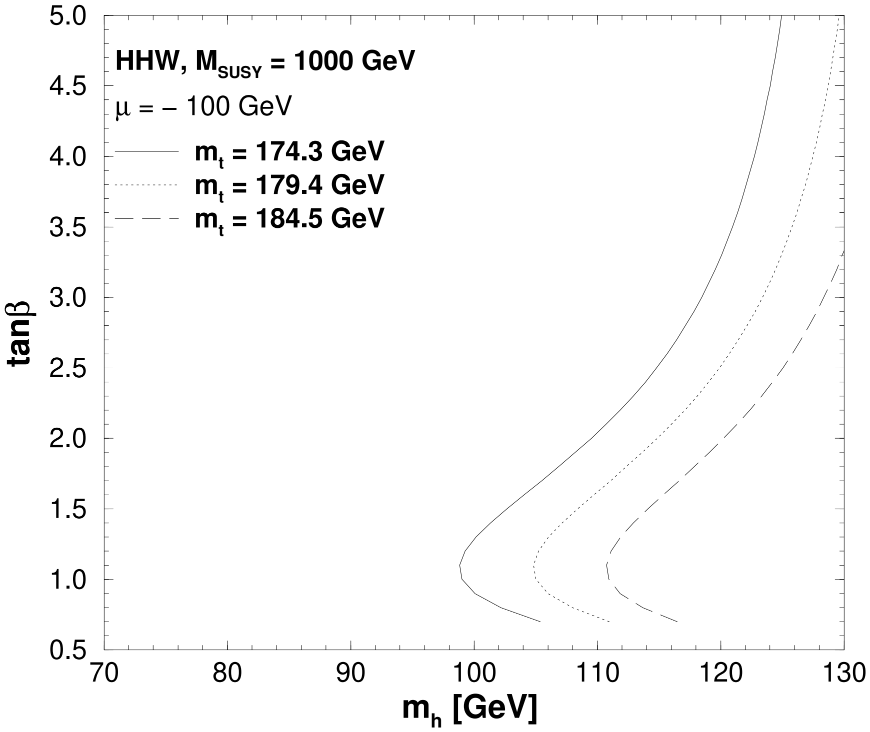

Figure 4: m_h is shown as a function of $\tan\beta$, evaluated in the FD approach. We give the results for three different values of the top-quark mass, $m_t = 174.3, 179.4, 184.5\,\mathrm{GeV}$.

The increase in the maximal value for m_h by about 4 GeV from the new FD result and by further 5 GeV if the benchmark scenario is slightly generalized has a significant effect on exclusion limits for $\tan\beta$ derived from the Higgs-boson search. Employing the benchmark scenario and the RG result, an excluded $\tan\beta$ range already appears for an experimental bound on m_h of slightly above 90 GeV, see figure 2. However, taking into account the above sources for an increase in the maximal value for m_h the current data (summer '99, see e.g. ref. [21]) from the Higgs-boson search hardly allow any $\tan\beta$ exclusion yet, see figure 3. Concerning the assumed m_h limit obtained at the end of LEP2, the accessible $\tan\beta$ region is largely reduced from the $m_h^{\max}$-RG to the $m_h^{\max}$-FD calculation.

3. Constraints on $\tan\beta$ "beyond the benchmark"

Since the dominant radiative corrections to the lightest Higgs-boson mass are proportional to m_t^4, the theoretical prediction for m_h depends sensitively on the precise value of the top-quark mass. The experimental uncertainty in the top-quark mass of currently $\Delta m_t = 5.1\,\mathrm{GeV}$ [22] thus has a strong effect on the prediction for the upper bound on m_h, where larger values of m_t give rise to larger values of m_h. An increase in m_t by $\Delta m_t = 5.1\,\mathrm{GeV}$ leads to an increase in m_h of up to 6 GeV, as shown in figure 4, where also the effect of increasing m_t by two standard deviations is displayed.

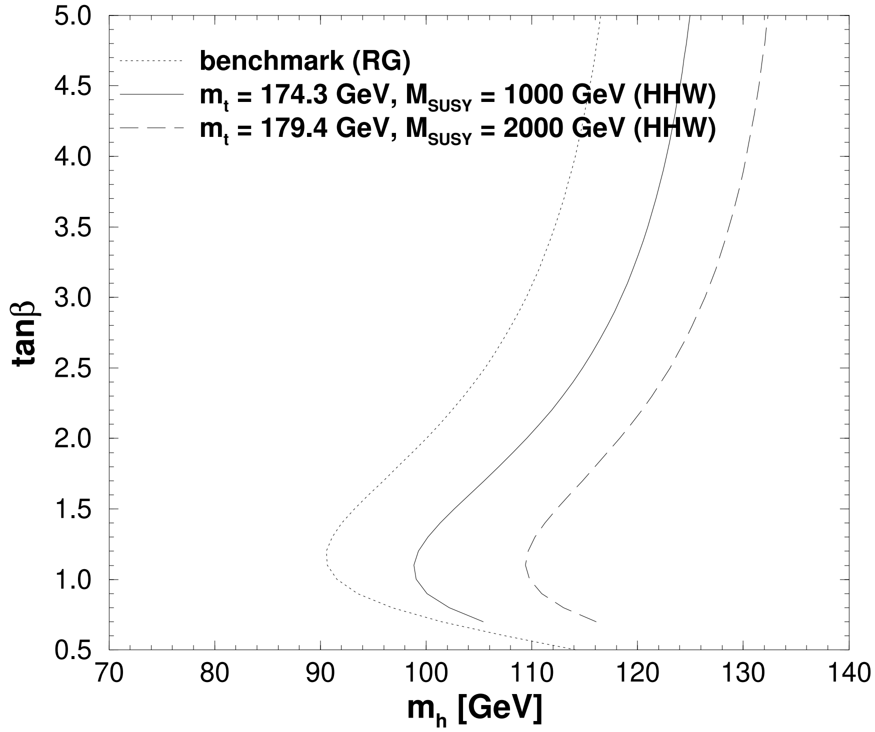

Figure 5: m_h is shown as a function of $\tan\beta$. The dotted curve displays the benchmark scenario in the RG approach, which has been used for phenomenological analyses up to now. The solid curve displays the $m_h^{\max}$-FD scenario, while the dashed curve corresponds to the "increased m_h" scenario with $m_t = 179.4\,\text{GeV}$ and $M_{\text{SUSY}} = 2000\,\text{GeV}$.

Besides the top-quark mass, the other main entry of the benchmark scenario is the choice $M_{\text{SUSY}} = 1\,\text{TeV}$. Similarly to the case of m_t, allowing for higher values of M_{SUSY} leads to higher values of m_h. Since M_{SUSY} enters only logarithmically in the prediction for m_h, the dependence on it is more moderate. An increase of M_{SUSY} from $1\,\text{TeV}$ to $2\,\text{TeV}$ enhances m_h by up to $4\,\text{GeV}$ (depending on $\tan\beta$).

Allowing values of m_t one or even two standard deviations above the current experimental central value and increasing also the input value of M_{SUSY} clearly has a large effect on possible $\tan\beta$ constraints. In figure 5 we show an "increased m_h" scenario, where $m_t = 179.4\,\text{GeV}$ has been chosen, i.e. one standard deviation above the current experimental value, and $M_{\text{SUSY}} = 2000\,\text{GeV}$ is taken. It is compared with the benchmark scenario in the RG calculation and with the $m_h^{\max}$-FD scenario. In the "increased m_h" scenario exclusion of a $\tan\beta$ range would become possible only with a limit on m_h of more than about $110\,\text{GeV}$.

In this context one should keep in mind that the benchmark scenario contains not only an assumption about the SUSY parameters but also about the actual model which is tested, namely a SUSY model with a minimal Higgs sector that does not contain $\mathcal{CP}$-violating phases. The upper bound on m_h, however, stays the same also with complex parameters [23]. Extensions of the Higgs sector by additional particle representations can shift the upper bound on the mass of the lightest Higgs boson up to values of about 200 GeV [24].

4. Conclusions

We have investigated the upper bound on the mass of the lightest $\mathcal{CP}$-even Higgs boson in the MSSM, depending on $\tan\beta$. In order to discuss possible exclusion limits on $\tan\beta$ from the direct Higgs-boson search, it is useful to consider definite scenarios with specific assumptions on the relevant input parameters and on the structure of the considered model. Constraints on $\tan\beta$ derived within such a framework are of course to be understood under the assumptions defining the investigated scenario.

In this spirit in particular the "benchmark" scenario has been widely used, in which $m_t = 175\,\mathrm{GeV}$ and $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$ are chosen. In this note we have analyzed the influence of variations in the other parameters entering the prediction for m_h and we have shown that the settings used for those parameters within the benchmark scenario do not cover the maximal possible value of m_h for $m_t = 175\,\mathrm{GeV}$ and $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$. We thus suggest a slight generalization of the definition of the benchmark scenario, where more general values of M_2 and M_A are allowed, a more conservative expression for the running top-quark mass is taken, and the case of maximal mixing in the scalar top sector is defined such that it corresponds to the maximal m_h value. Compared to the definition of the benchmark scenario used so far, the generalization suggested here leads to a shift in the upper bound of m_h of about 5 GeV.

Independently of the precise definition of the benchmark scenario, we have furthermore analyzed the impact of taking into account the new diagrammatic two-loop result (program FeynHiggs) for the mass of the lightest Higgs boson, which contains in particular genuine non-logarithmic two-loop contributions that are not present in the previous result obtained by renormalization group methods (program subhpole). The maximal value for m_h obtained with FeynHiggs is higher by about 4 GeV than the maximal value calculated with subhpole. This leads to a significant reduction of the $\tan\beta$ region accessible at LEP2.

Going beyond the benchmark scenario, we have also discussed an "increased m_h" scenario, where m_t is chosen to be one standard deviation above the current experimental central value and $M_{\mathrm{SUSY}} = 2\,\mathrm{TeV}$. In this scenario no values of $\tan\beta$ can be excluded as long as the limit on m_h is lower than about 110 GeV.

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